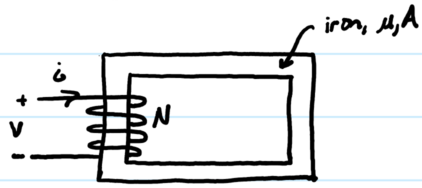
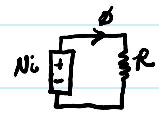
# 2018-09-19-1

## Last time:




$\mathcal{R} = \frac{\ell}{\mu A}$

$\mathcal{R}\phi = Ni$

$\phi = BA$

$\mu H = B$

$\lambda = N\phi$

$L = \frac{\mu A N^2}{\ell}$

$\lambda = Li$

$V = \frac{d\lambda}{dt} \Rightarrow V = L\frac{di}{dt}$

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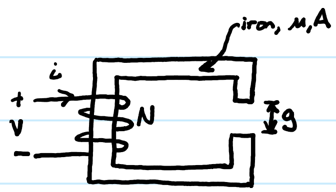
Today: 1) Multiple Reluctances

2) Fringing effects

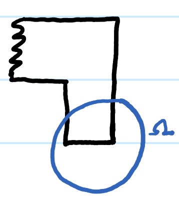
* Add air gap into iron core




$$\oint_C \underline{H} \cdot \underline{d\ell} = \int_{\partial\Omega} \underline{J_f} \cdot \hat{n}\, dA$$

$H_{iron}\ell + H_g g = Ni$ ($\ell$ is the mean path length inside iron)

Conservation of flux




$$\oint_{\partial\Omega} \underline{B} \cdot \hat{n}\, dA = 0$$

$B_g A_g - B_{iron} A_{iron} = 0$

$B_g A_g = B_{iron} A_{iron}$

*Assume No fringing: $A_g = A_{iron}$

$B_g = B_{iron}$

Linear relations: $\mu H_{iron} = B_{iron}$ $\quad$ $\mu_0 H_g = B_g$

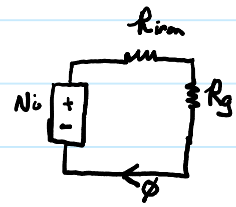
$H_{iron} = \frac{B_{iron}}{\mu}$ $\quad$ $H_g = \frac{B_g}{\mu_0}$

$$B_{iron}\left(\frac{\ell}{\mu}\right) + B_g\left(\frac{g}{\mu_0}\right) = Ni$$

$\phi = B_{iron} A_{iron} = B_g A_g$

$$\phi\left(\frac{l}{\mu A_{iron}}\right) + \phi\left(\frac{g}{\mu_0 A_g}\right) = Ni$$

$$\phi\left(\mathcal{R}_{iron} + \mathcal{R}_g\right) = Ni$$




* Note: $\mathcal{R}_{iron} \ll \mathcal{R}_g$

$$\boxed{\phi = \frac{Ni}{\mathcal{R}_{iron} + \mathcal{R}_g}}$$

<u>Faraday's law</u>

$$\oint_c \underline{E} \cdot d\underline{l} = -\frac{d}{dt}\left(\int_{\partial\Omega} \underline{B} \cdot \hat{n}\, dA\right)$$

$$-V + 0 = -\frac{d}{dt}(NBA) \Rightarrow V = \frac{d}{dt}(N\phi)$$

$$\boxed{V = \frac{d\lambda}{dt}}$$

$$\lambda = N\phi \Rightarrow \lambda = \left(\frac{N^2}{\mathcal{R}_{iron} + \mathcal{R}_g}\right) i$$

$$\boxed{V = L\frac{di}{dt}}$$

$$L = \frac{N^2}{\mathcal{R}_{iron} + \mathcal{R}_g} \Rightarrow \lambda = Li$$

* If $\mu \to \infty$, $\mathcal{R}_{iron} \to 0$ $\Rightarrow L \approx \dfrac{N^2}{\mathcal{R}_g} \Rightarrow L \approx \dfrac{\mu_0 A_g N^2}{g}$

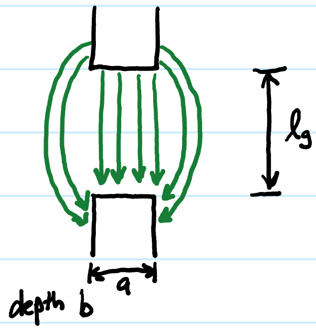
* What does $\mu \to \infty$ mean?

$B = \mu H \Rightarrow H = \frac{B}{\mu}$

$B$ is finite from conservation of flux. Therefore, as $\mu \to \infty$, $H \to 0$

$\mathcal{R} \to 0$ $\left(\mathcal{R} \sim \frac{1}{\mu}\right)$

* How would fringing affect results? (Fringing: flux spreads out of gap into surrounding air)




$A_{core} = ab$

$A_g = (a + l_g)(b + l_g)$ [*Note: empirical relation]

$= ab + l_g(a+b) + l_g^2$

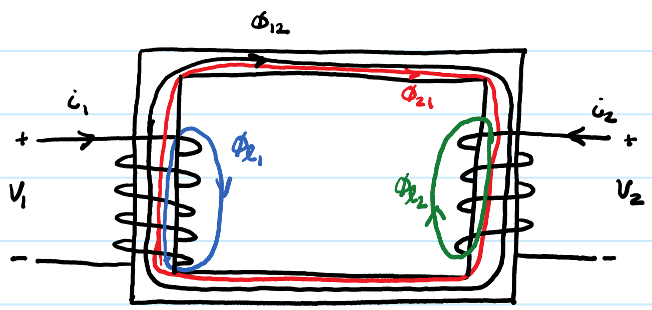
$A_g = ab\left(1 + l_g\left(\frac{1}{b} + \frac{1}{a}\right) + \frac{l_g^2}{ab}\right)$

$$\boxed{A_g = A_{core}\left(1 + l_g\left(\frac{1}{b} + \frac{1}{a}\right) + \frac{l_g^2}{ab}\right)}$$

* in $\lim_{l_g \to 0} A_g = A_c$

* Therefore, Fringing is minimized by lowering $l_g$.

## Mutual Inductance:




$\phi_{12}$: flux in coil 1 due to $i_2$

$\phi_{21}$: flux in coil 2 due to $i_1$

$\phi_{l_1}$: leakage flux from coil 1

$\phi_{l_2}$: leakage flux from coil 2

Total flux:

$\phi_1 = \phi_{l_1} + \phi_{21} + \phi_{12} \Rightarrow \phi_1 = \phi_{11} + \phi_{12}$

$\phi_2 = \phi_{21} + \phi_{l_2} + \phi_{12} \Rightarrow \phi_2 = \phi_{21} + \phi_{22}$

$\phi_{11} = \phi_{l_1} + \phi_{21}$ : total flux produced by coil 1 only

$\phi_{22} = \phi_{l_2} + \phi_{12}$ : total flux produced by coil 2 only

**Flux linkages:**

$$\lambda_1 = N_1\phi_1 \Rightarrow \lambda_1 = N_1\phi_{11} + N_1\phi_{12}$$
$$\lambda_2 = N_2\phi_2 \Rightarrow \lambda_1 = N_2\phi_{21} + N_2\phi_{22}$$

$N_1\phi_1 = L_1 i_1$ from before. $L_1$ is self inductance of coil 1

$N_2\phi_2 = L_2 i_2$ from before. $L_2$ is self inductance of coil 2

$N_1\phi_{12} = M i_2$

$N_2\phi_{21} = M i_1$

$M$ is the mutual inductance between the coils

**coefficient of coupling:**

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$0 \le k \le 1 \Rightarrow k=0$: No coupling

$k=1$: perfect coupling

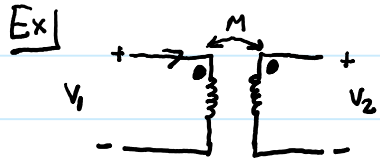
$$\boxed{\begin{aligned}\lambda_1 &= L_1 i_1 + M i_2 \\ \lambda_2 &= M i_1 + L_2 i_2\end{aligned}}$$

$$\boxed{\begin{aligned}V_1 &= L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \\ V_2 &= M\frac{di_2}{dt} + L_2\frac{di_1}{dt}\end{aligned}}$$

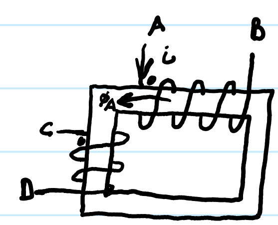
**Polarity markings (dot convention)**

* Changing magnetic flux induces a voltage (Lenz's law)
* Note polarity using dots where current entering a dotted terminal in one winding induces a voltage with positive polarity at the dotted terminal of the other winding.

Ex

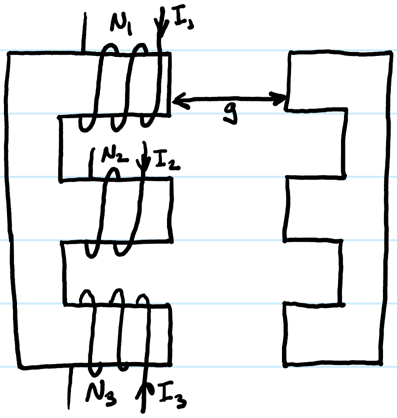



Ex




**Steps to determine:**
1) Inject current at first dot and use RHR to determine flux direction
2) Put second dot on terminal of second coil to give same flux direction

Ex



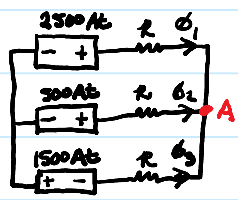

**Given:** $N_1 = 100$, $N_2 = 50$, $N_3 = 100$, $I_1 = 25A$, $I_2 = 10A$, $I_3 = 15A$, $A_g = 4cm^2$, $\mu = \infty$, $g = 0.1cm$

**Find:** Flux through the coils

**Solution:** All air gaps are the same

$$\mathcal{R}_1 = \mathcal{R}_2 = \mathcal{R}_3 = \mathcal{R} = \frac{g}{\mu_0 A} = \frac{(0.001m)}{(4\pi \times 10^{-7}\ H/m)(.0004m^2)} \Rightarrow \mathcal{R} = 1.99 \times 10^6\ At/Wb$$

**Magnetic Circuit:**




at point A: $\phi_1 + \phi_2 + \phi_3 = 0$

Let the MMF at A be $f$

$$\phi_1 = \frac{2500 - f}{\mathcal{R}} \qquad \phi_2 = \frac{500 - f}{\mathcal{R}} \qquad \phi_3 = \frac{-1500 - f}{\mathcal{R}}$$

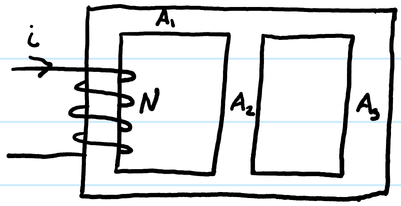
$$\frac{2500 - f}{\mathcal{R}} + \frac{500 - f}{\mathcal{R}} - \frac{(1500 + f)}{\mathcal{R}} = 0 \Rightarrow 3000 - 2f - 1500 - f = 0$$

$$1500 - 3f = 0$$

$$\boxed{f = 500At}$$

$$\boxed{\begin{aligned} \phi_1 &= 1 \times 10^{-3}\ Wb \\ \phi_2 &= 0\ Wb \\ \phi_3 &= -1 \times 10^{-3}\ Wb \end{aligned}}$$

Ex



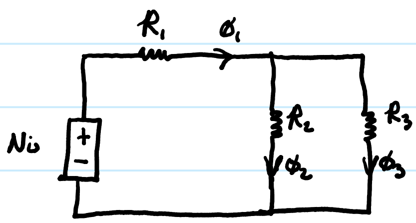

Given: $\mu = \mu_o \mu_r$ $\quad N = 25$ $\quad i = 0.5A$

$\mu_1 = \mu_3 = 2250$ $\quad l_1 = l_3 = 30cm$ $\quad A_1 = A_3 = 2cm^2$

$\mu_2 = 1350$ $\quad l_2 = 10cm$ $\quad A_2 = 4cm^2$

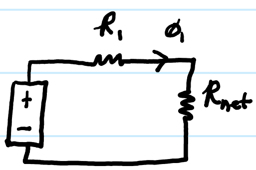
Find: $B_1, B_2, B_3$

Solution: Magnetic circuit




$\mathcal{R}_1 = \mathcal{R}_3 = 5.31 \times 10^5 \ At/Wb$

$\mathcal{R}_2 = 1.47 \times 10^5 \ At/Wb$




$$\frac{1}{\mathcal{R}_{net}} = \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3}$$

$\mathcal{R}_{net} = 1.15 \times 10^5 \ At/Wb$

$$\phi_1 = \frac{Ni}{\mathcal{R}_1 + \mathcal{R}_{net}} \Rightarrow \phi_1 = B_1 A_1 = 19.35 \times 10^{-6} \ Wb$$

$$\boxed{B_1 = 0.09675 \ Wb/m^2}$$

$Ni - \mathcal{R}_1 \phi_1 = \mathcal{F}$

$\mathcal{F} = 2.23 \ At$

$$\phi_2 = \frac{\mathcal{F}}{\mathcal{R}_2} \Rightarrow \phi_2 = B_2 A_2 = 15.17 \times 10^{-6} \ Wb$$

$$\boxed{B_2 = 0.0379 \ Wb/m^2}$$

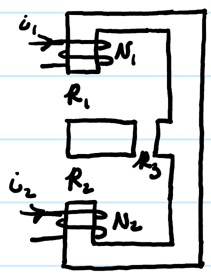
$\phi_1 = \phi_2 + \phi_3$

$\phi_3 = \phi_1 - \phi_2 \Rightarrow \phi_3 = B_3 A_3 = 4.18 \times 10^{-6} \ Wb$

$$\boxed{B_3 = 0.0209 \ Wb/m^2}$$

# 2018-09-19-7

**Ex**



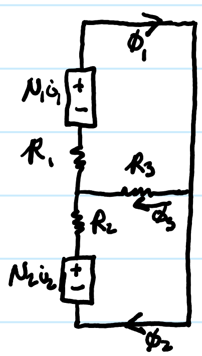

$R_1 = 3\times10^6$ At/Wb $\qquad N_1 = 100$

$R_2 = 2\times10^6$ At/Wb $\qquad N_2 = 100$

$R_3 = 2\times10^6$ At/Wb

**Find:** $L_1, L_2, M$

**Solution:** Magnetic Circuit




$$\phi_1 = \phi_2 + \phi_3 \Rightarrow \phi_3 = \phi_1 - \phi_2$$

$$N_1 i_1 = R_1\phi_1 + R_3\phi_3 \qquad N_2 i_2 = R_2\phi_2 - R_3\phi_3$$

$$N_1 i_1 = R_1\phi_1 + R_3(\phi_1 - \phi_2) \qquad N_2 i_2 = R_2\phi_2 - R_3(\phi_1 - \phi_2)$$

$$N_1 i_1 = \phi_1(R_1 + R_3) - R_3\phi_2 \qquad N_2 i_2 = (R_2 + R_3)\phi_2 - R_3\phi_1$$

$$\phi_1 = \frac{N_1 i_1 + R_3\phi_2}{R_1 + R_3} \qquad N_2 i_2 = (R_2 + R_3)\phi_2 - \frac{R_3 N_1 i_1}{R_1 + R_3} - \frac{R_3^2}{R_1 + R_3}\phi_2$$

$$\boxed{\phi_1 = 25\times10^{-6} i_1 + 12.5\times10^{-6} i_2 \text{ Wb}}$$

$$N_2 i_2 + \left(\frac{R_3}{R_1 + R_3}\right) N_1 i_1 = \phi_2\left[R_2 + R_3 - \frac{R_3^2}{R_1 + R_3}\right]$$

$$\phi_2 = \frac{1}{R_2 + R_3 - \frac{R_3^2}{R_1 + R_3}}\left(\left(\frac{R_3}{R_1 + R_3}\right) N_1 i_1 + N_2 i_2\right)$$

$$\boxed{\phi_2 = 12.5\times10^{-6} i_1 + 31.25\times10^{-6} i_2 \text{ Wb}}$$

$$\lambda_1 = N_1\phi_1 \Rightarrow \lambda_1 = 25\times10^{-4} i_1 + 12.5\times10^{-4} i_2 \text{ Wbt}$$

$$\lambda_2 = N_2\phi_3 \Rightarrow \lambda_2 = 12.5\times10^{-4} i_2 + 31.25\times10^{-4} \text{ Wbt}$$

$$\boxed{\begin{aligned} L_1 &= 25\times10^{-4} \text{ H} \\ L_2 &= 31.25\times10^{-4} \text{ H} \\ M &= 12.5\times10^{-4} \text{ H} \end{aligned}}$$